Math 323 - Formal Mathematical Reasoning and Writing

Problem Session

Wednesday, 4/22/15

Images and Preimages

1. [1] Let $f : A \to b$ be an injective function. Then if S_i with $i \in \mathcal{I}$ is a family of sets where $\forall i \in \mathcal{I}, S_i \subseteq A$, then
$$f\left(\bigcap_{i \in \mathcal{I}} S_i\right) = \bigcap_{i \in I} f(S_i)$$

Infinity!!

1. Let L be the line $y = rx$, where r is a rational number. Let A be the set of all points (a, b) such that $a, b \in \mathbb{Z}$ and (a, b) is on the line L. Prove that A is infinite.
2. Can you think of a *bijective* function f with $f : \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$? What does this tell you about the cardinality of the set $(-\frac{\pi}{2}, \frac{\pi}{2})$? Can you edit the function you came up with to provide a bijection $f_1 : \mathbb{R} \to (0, 1)$?
3. Prove that the $\mathbb{N}$ has the same cardinality as $\mathbb{N} \times \mathbb{N}$ by describing a bijection between the two sets.

[1] Madden §12.3 #6